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Symmetry properties in polarimetric remote sensing

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This paper presents the relations among polarimetric backscattering coefficients from the viewpoint of symmetry groups. Symmetry of geophysical media encountered in remote sensing due to reflection, rotation, azimuthal, and centrical symmetry groups is considered for both reciprocal and nonreciprocal cases. On the basis of the invariance under symmetry transformations in the linear polarization basis, the scattering coefficients are related by a set of equations which restrict the number of independent parameters in the polarimetric covariance matrix. The properties derived under these transformations are general and apply to all scattering mechanisms in a given symmetrical configuration. The scattering coefficients calculated from theoretical models for layer random media and rough surfaces are shown to obey the derived symmetry relations. Use of symmetry properties in remote sensing of structural and environmental responses of scattering media is discussed. As a practical application, the results from this paper provide new methods for the external calibration of polarimetric radars without the deployment of man-made calibration targets.

1. INTRODUCTION

In geophysical remote sensing, the encountered media usually possess symmetry properties. For instance, the ocean exhibits symmetry about the downwind or upwind direction [*Stewart*, 1985]. Multiyear sea ice has embedded air bubbles and a hummocky topography [*Weeks and Ackley*, 1982] which do not show any azimuthal preference. The canopy layer of a natural forest consists of leaves, twigs, and branches randomly oriented in azimuthal directions [*Kimes et al.*, 1979]. Polarimetric remote sensing data have been collected extensively, and theoretical models have been developed intensively for understanding the responses of these geophysical media. It is therefore useful to investigate the symmetry properties of these media for applications in both theoretical and practical problems.

Polarimetric backscattering properties of the media can be described completely with a set of scattering coefficients in the covariance matrix [*Nghiem et al.*, 1990]. From the symmetry of the media, some restrictions on the scattering coefficients have been recognized. Up to the second-order Born approximation it has been shown that the scattering coefficients, which correspond to the correlation between the copolarized and cross-polarized elements of the scattering matrix, are zero for volume scattering from layer random media containing spherical scatterers [*Borgeaud et al.*, 1987]. The same result has been obtained for scattering from inhomogeneous layer media consisting of nonspherical scatterers with random azimuthal orientations under the first-order distorted Born approximation [*Nghiem*, 1991]. These zero-scattering coefficients have been assumed to extract equations relating true and measured quantities for use in the calibration of polarimetric scattering data [*Yueh et al.*, 1991; *van Zyl*, 1990; *Sheen et al.*, 1989] and to study the unpolarized component of the scattering from a forested area [*Durden et al.*, 1990].

Generally, the question is how symmetries of the media manifest themselves in the polarimetric backscattering coefficients. Specifically, is it possible to prove that the scattering coefficients, which correlate the copolarized and cross-polarized elements in the scattering matrix of a symmetrical medium, are zero without any restriction to the mathematical orders of scattering or the scattering mechanisms themselves? Is the azimuthal symmetry an overrequirement for those scattering coefficients to be zero? Furthermore, is it possible to derive new relationships between the scattering coefficients from the symmetry properties? As a systematic approach, symmetry groups are considered. Symmetries have been defined and mathematically formulated in group theory and applied to various

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fields of physics [*Hamermesh*, 1972]. The symmetry of a body is described by a set of transformations which preserve distances. These transformations can be constructed from three fundamental operations: mirror reflection, axial rotation, and linear translation. The translational invariance of the covariance matrix, implied in the subsequent development in this paper, in the horizontal direction is usually observed for a geophysical medium whose scattering matrix is statistically homogeneous within the distributed extent. More interestingly, the reflection, the rotation, and their combinations constitute symmetry groups applicable to geophysical remote sensing of media with reflection, rotation, azimuthal, and centrical symmetries.

In this paper the invariance of backscattering coefficients in the covariance matrix under the symmetry transformations of the linear polarization basis is used to investigate the conditions imposed by the aforementioned symmetries for both reciprocal and nonreciprocal media. The paper is organized into eight sections. In section 2 the rotation of scattering coefficients in a linear polarization basis is carried out. Section 3 considers mirror reflection symmetry about a plane to generally prove the complete decorrelation between the copolarized and the cross-polarized scattering elements resulting in the corresponding zero cross-scattering coefficients. In this case, theoretical results for scattering from a random medium with aligned spheroidal scatterers and from a randomly perturbed periodic rough surface are shown to satisfy this symmetry condition. Section 4 studies the consequence of two-dimensional pure rotation symmetry about an axis on the polarimetric scattering coefficients. Because of this rotation invariance the scattering coefficients are constrained under a new set of equations which hold true even for gyrotropic and chiral media. Section 5 analyzes the implications of azimuthal symmetry. This symmetry group is obtained from the rotation group by adjoining the reflection in the vertical plane containing the axis of the rotation symmetry. The azimuthal symmetry interrelates the scattering coefficients with a set of equations which are used to test the volume scattering from layer random media under the first-order distorted Born approximation and the rough surface scattering under the first-order small perturbation method. Section 6 examines centrical symmetry about a point. This symmetry can be considered as azimuthal symmetry with the axis containing the center point and rotated in three dimensions. Theoretical calculations of volume scattering under the first-order distorted Born approximation for a layer of randomly oriented spheroidal scatterers and surface scattering under the geometrical optics approximation for a rough interface are shown to abide by the constraints from centrical symmetry. Section 7 discusses the use of symmetry properties in polarimetric remote sensing of medium structures and environmental effects and suggests new methods for the calibration of polarimetric radars. Finally, the paper is summarized in section 8.

2. SCATTERING COEFFICIENTS

In this section the scattering coefficients in a rotated linear polarization basis are calculated in terms of those in the original basis. Consider an electric field $\bar{E}_i = (\hat{h}E_{hi} + \hat{v}E_{vi})$ incident on a scattering medium giving rise to the scattered field $\bar{E}_s = (\hat{h}E_{hs} + \hat{v}E_{vs})$, where h represents the horizontal polarization and v represents the vertical polarization. The scattered field is related to the incident field by a scattering matrix **F** defined as [*Nghiem et al.*, 1990]

$$\begin{bmatrix} E_{hs} \\ E_{vs} \end{bmatrix} = \frac{e^{ikr}}{r}\mathbf{F}\begin{bmatrix} E_{hi} \\ E_{vi} \end{bmatrix} = \frac{e^{ikr}}{r}\begin{bmatrix} f_{hh} & f_{hv} \\ f_{vh} & f_{vv} \end{bmatrix}\begin{bmatrix} E_{hi} \\ E_{vi} \end{bmatrix} \tag{1}$$

where factor e^{ikr}/r is the spherical wave transformation, scattering element $f_{\mu\nu}$ consists of scattered polarization μ and incident polarization ν with μ and ν being of h or v, and the incident basis has been applied to both incident and scattered fields.

The elements of the scattering matrix can be used to find the backscattering coefficients determined by

$$\sigma_{\mu\tau\nu\kappa} = \lim_{\substack{r\to\infty \\ A\to\infty}} \frac{4\pi r^2}{A} \frac{\langle E_{\mu s}E^*_{\nu s}\rangle}{E_{\tau i}E^*_{\kappa i}} \tag{2}$$

where subscripts μ, ν, τ, and κ can be h or v and A is the illuminated area. The scattered fields in (2) are obtained by measuring h and v returns, while the incident field is transmitted exclusively with h or v polarization. As seen from (1), this measurement procedure is mathematically described by the following equations:

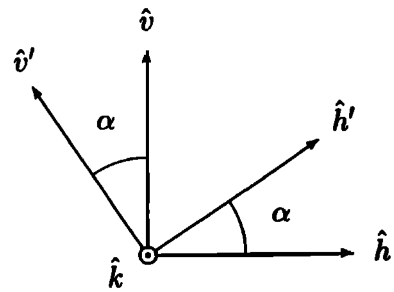

Fig. 1. Rotation of the linear polarization basis by angle α.

$$E_{\mu s} = \frac{e^{ikr}}{r}\,(f_{\mu\tau}E_{\tau i} + f_{\mu\kappa}E_{\kappa i})|_{E_{\kappa i}=0} = \frac{e^{ikr}}{r} f_{\mu\tau}E_{\tau i} \quad (3a)$$

$$E_{\nu s} = \frac{e^{ikr}}{r}\,(f_{\nu\tau}E_{\tau i} + f_{\nu\kappa}E_{\kappa i})|_{E_{\tau i}=0} = \frac{e^{ikr}}{r} f_{\nu\kappa}E_{\kappa i} \quad (3b)$$

Introducing (3) in (2) renders the polarimetric backscattering coefficient $\sigma_{\mu\nu\tau\kappa}$ in terms of the scattering matrix elements:

$$\sigma_{\mu\tau\nu\kappa} = \lim_{A\to\infty} \frac{4\pi}{A}\langle f_{\mu\tau} f^*_{\nu\kappa}\rangle \quad (4)$$

Since there are four elements in the scattering matrix, the full covariance matrix is a four-by-four matrix composed of 16 scattering coefficients

$$\mathbf{C} = \begin{bmatrix} \sigma_{hhhh} & \sigma_{hhhv} & \sigma_{hhvh} & \sigma_{hhvv} \\ \sigma_{hvhh} & \sigma_{hvhv} & \sigma_{hvvh} & \sigma_{hvvv} \\ \sigma_{vhhh} & \sigma_{vhhv} & \sigma_{vhvh} & \sigma_{vhvv} \\ \sigma_{vvhh} & \sigma_{vvhv} & \sigma_{vvvh} & \sigma_{vvvv} \end{bmatrix}$$

$$= \begin{bmatrix} \sigma_{hhhh} & \sigma_{hhhv} & \sigma_{hhvh} & \sigma_{hhvv} \\ \sigma^*_{hhhv} & \sigma_{hvhv} & \sigma_{hvvh} & \sigma_{hvvv} \\ \sigma^*_{hhvh} & \sigma^*_{hvvh} & \sigma_{vhvh} & \sigma_{vhvv} \\ \sigma^*_{hhvv} & \sigma^*_{hvvv} & \sigma^*_{vhvv} & \sigma_{vvvv} \end{bmatrix} \quad (5)$$

In this covariance matrix the four diagonal elements σ_{hhhh}, σ_{hvhv}, σ_{vhvh}, and σ_{vvvv} are the conventional backscattering coefficients σ_{hh}, σ_{hv}, σ_{vh}, and σ_{vv}, respectively, which are real quantities. In general, the off-diagonal elements are complex. Thus the covariance matrix contains at most 16 independent parameters.

Let $\hat{h}$ rotate to $\hat{h}'$ and $\hat{v}$ to $\hat{v}'$ by an angle α around the incident direction $\hat{k}$ in the counterclockwise sense, as shown in Figure 1. The rotated polarization vectors $\hat{h}'$ and $\hat{v}'$ are related to the original vectors as

$$\begin{bmatrix} \hat{h}' \\ \hat{v}' \end{bmatrix} = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \hat{h} \\ \hat{v} \end{bmatrix} \quad (6)$$

To obtain the elements of the scattering matrix in the rotated linear basis, the measurement procedure (3) is utilized to obtain

$$E'_{\mu s} = \frac{e^{ikr}}{r}\,\hat{\mu}'^{\top}\cdot\mathbf{F}\cdot\hat{\tau}'E'_{\tau i} = \frac{e^{ikr}}{r} f'_{\mu\tau}E'_{\tau i}$$

$$\Rightarrow f'_{\mu\tau} = \hat{\mu}'^{\top}\cdot\mathbf{F}\cdot\hat{\tau}' \quad (7)$$

where $\top$ denotes the transpose and $\hat{\mu}'$ or $\hat{\tau}'$ can be $\hat{h}'$ or $\hat{v}'$. Explicitly, the elements of the new scattering matrix written as a function of the rotation angle α are

$$f'_{hh} = \cos\alpha\,(f_{hh}\cos\alpha + f_{hv}\sin\alpha)$$
$$+ \sin\alpha\,(f_{vh}\cos\alpha + f_{vv}\sin\alpha) \quad (8a)$$

$$f'_{hv} = \cos\alpha\,(-f_{hh}\sin\alpha + f_{hv}\cos\alpha)$$
$$+ \sin\alpha\,(-f_{vh}\sin\alpha + f_{vv}\cos\alpha) \quad (8b)$$

$$f'_{vh} = -\sin\alpha\,(f_{hh}\cos\alpha + f_{hv}\sin\alpha)$$
$$+ \cos\alpha\,(f_{vh}\cos\alpha + f_{vv}\sin\alpha) \quad (8c)$$

$$f'_{vv} = -\sin\alpha\,(-f_{hh}\sin\alpha + f_{hv}\cos\alpha)$$
$$+ \cos\alpha\; -f_{vh}\sin\alpha + f_{vv}\cos\alpha) \quad (8d)$$

Substituting (8) in (4) yields the scattering coefficients in the rotated linear polarization basis as linear combinations of the original scattering coefficients. The complete set of the elements in the new covariance matrix are shown in the appendix.

The four-by-four covariance matrix describes the polarimetric backscattering properties of both reciprocal and nonreciprocal media. For reciprocal media the reciprocity relation $f_{hv} = f_{vh}$ reduces the covariance matrix to a three-by-three matrix [*Nghiem et al.*, 1990]

$$\mathbf{C} = \begin{bmatrix} \sigma_{hhhh} & \sigma_{hhhv} & \sigma_{hhvv} \\ \sigma^*_{hhhv} & \sigma_{hvhv} & \sigma_{hvvv} \\ \sigma^*_{hhvv} & \sigma^*_{hvvv} & \sigma_{vvvv} \end{bmatrix}$$

$$= \sigma \begin{bmatrix} 1 & \beta(e)^{1/2} & \rho(\gamma)^{1/2} \\ \beta^*(e)^{1/2} & e & \xi(\gamma e)^{1/2} \\ \rho^*(\gamma)^{1/2} & \xi^*(\gamma e)^{1/2} & \gamma \end{bmatrix} \quad (9)$$

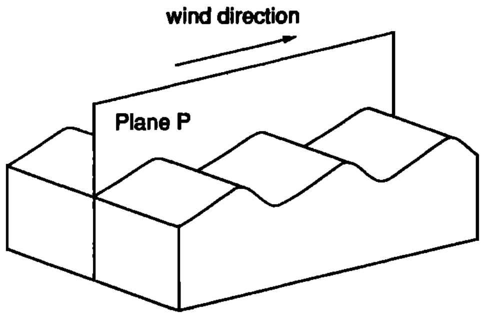

Fig. 2. Reflection symmetry of water waves about a vertical plane (P) parallel to the downwind or upwind direction.

where the normalizing factor is $\sigma = \sigma_{hhhh}$ and the intensity ratios γ and e and the correlation coefficients ρ, β, and ξ are defined as

$$\gamma = \frac{\sigma_{vvvv}}{\sigma}, \qquad e = \frac{\sigma_{hvhv}}{\sigma} \tag{10a}$$

$$\rho = \frac{\sigma_{hhvv}}{\sigma(\gamma)^{1/2}}, \qquad \beta = \frac{\sigma_{hhhv}}{\sigma(e)^{1/2}}, \qquad \xi = \frac{\sigma_{hvvv}}{\sigma(\gamma e)^{1/2}} \tag{10b}$$

It is also observed from (8b) and (8c) that $f_{hv} = f_{vh}$ implies $f'_{hv} = f'_{vh}$. The reciprocity relation is therefore invariant under the rotation of the linear polarization basis. The results in this section are used in the subsequent sections to analyze the effects of symmetry properties on polarimetric backscattering coefficients.

3. REFLECTION SYMMETRY

In this section the reflection symmetry with respect to a vertical plane is considered in order to find the constraints on the scattering coefficients. This symmetry group has the mirror reflection transformation denoted by σ_v, where the subscript v stands for vertical (not to be confused with the conventional scattering coefficient $\sigma_{vv} = \sigma_{vvvv}$) in the notation of group theory [*Hamermesh*, 1972]. On the ocean surface, for instance, water waves have the reflection symmetry about a vertical plane (P) parallel to the downwind or upwind direction as depicted in Figure 2. Here theoretical results for the scattering from a random medium with aligned ellipsoidal scatterers and from a randomly perturbed periodic rough surface will be shown to obey the constraints imposed by reflection symmetry.

Let the linear polarization basis ($\hat{h}$, $\hat{v}$) be oriented such that $\hat{h} \perp P$ and $\hat{v} \parallel P$. Then, the reflection symmetry requires that the measurements made by transmitting v or h and receiving v in the linear polarization basis rotated by $\alpha + (\pi/2)$ to be the same as the measurements made by transmitting h or v and receiving h in the basis rotated by $-\alpha$ as illustrated in Figure 3. This signifies

$$\sigma'_{vvvh}(\alpha + \pi/2) = \sigma'_{hhhv}(-\alpha) \tag{11}$$

Written in terms of angle α, the expressions for $\sigma'_{vvvh}(\alpha + \pi/2) = \sigma'^{*}_{vhvv}(\alpha + \pi/2)$ from (A9) in the appendix and $\sigma'_{hhhv}(-\alpha)$ from (A2) are

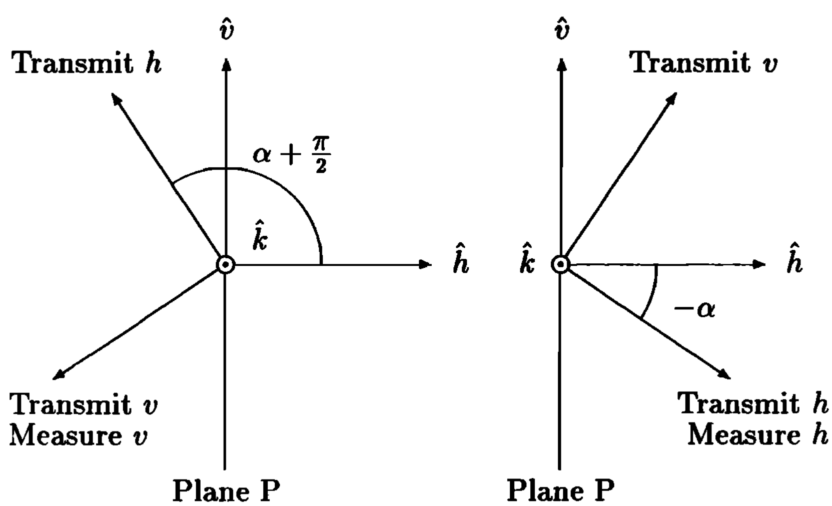

Fig. 3. Reflection symmetry in the measurements made by transmitting v or h and receiving v in the linear polarization basis rotated by $\alpha + (\pi/2)$ and by transmitting h or v and receiving h in the basis rotated by $-\alpha$.

$$\sigma'^*_{vhvv}\left(\alpha + \frac{\pi}{2}\right) = \sigma_{hhhh} \cos^3 \alpha \sin \alpha$$
$$+ \sigma^*_{hhhv} \cos^2 \alpha \sin^2 \alpha + \sigma^*_{hhvh} \cos^2 \alpha \sin^2 \alpha$$
$$+ \sigma^*_{hhvv} \cos \alpha \sin^3 \alpha - \sigma_{hhhv} \cos^4 \alpha$$
$$- \sigma_{hvhv} \cos^3 \alpha \sin \alpha - \sigma^*_{hvvh} \cos^3 \alpha \sin \alpha$$
$$- \sigma^*_{hvvv} \cos^2 \alpha \sin^2 \alpha + \sigma_{hhvh} \cos^2 \alpha \sin^2 \alpha$$
$$+ \sigma_{hvvh} \sin^3 \alpha \cos \alpha + \sigma_{vhvh} \cos \alpha \sin^3 \alpha$$
$$+ \sigma^*_{vhvv} \sin^4 \alpha - \sigma_{hhvv} \cos^3 \alpha \sin \alpha$$
$$- \sigma_{hvvv} \cos^2 \alpha \sin^2 \alpha - \sigma_{vhvv} \cos^2 \alpha \sin^2 \alpha$$
$$- \sigma_{vvvv} \sin^3 \alpha \cos \alpha \qquad (12a)$$

$$\sigma'_{hhhv}(-\alpha) = \sigma_{hhhh} \cos^3 \alpha \sin \alpha + \sigma_{hhhv} \cos^4 \alpha$$
$$- \sigma_{hhvh} \cos^2 \alpha \sin^2 \alpha - \sigma_{hhvv} \cos^3 \alpha \sin \alpha$$
$$+ \sigma^*_{hhhv} \cos^2 \alpha \sin^2 \alpha - \sigma_{hvhv} \cos^3 \alpha \sin \alpha$$
$$+ \sigma_{hvvh} \cos \alpha \sin^3 \alpha + \sigma_{hvvv} \cos^2 \alpha \sin^2 \alpha$$
$$- \sigma^*_{hhvh} \cos^2 \alpha \sin^2 \alpha - \sigma^*_{hvvh} \cos^3 \alpha \sin \alpha$$
$$+ \sigma_{vhvh} \cos \alpha \sin^3 \alpha + \sigma_{vhvv} \cos^2 \alpha \sin^2 \alpha$$
$$+ \sigma^*_{hhvv} \cos \alpha \sin^3 \alpha + \sigma^*_{hvvv} \cos^2 \alpha \sin^2 \alpha$$
$$- \sigma^*_{vhvv} \sin^4 \alpha - \sigma_{vvvv} \cos \alpha \sin^3 \alpha \qquad (12b)$$

Equating (12a) and (12b) in accordance with (11) enforces the following conditions on the real part (Re) and the imaginary part (Im) of the scattering coefficients:

$$2(\mathrm{Re}\ \sigma_{vhvv} + \mathrm{Re}\ \sigma_{hvvv} - \mathrm{Re}\ \sigma_{hhhv}$$
$$- \mathrm{Re}\ \sigma_{hhvh}) \sin^2 \alpha \cos^2 \alpha + \mathrm{Re}\ \sigma_{hhhv} \cos^2 \alpha$$
$$- \mathrm{Re}\ \sigma_{vhvv} \sin^2 \alpha = 0 \qquad (13a)$$

$$\mathrm{Im}\ \sigma_{hhhv} \cos^2 \alpha + \mathrm{Im}\ \sigma_{vhvv} \sin^2 \alpha = 0 \qquad (13b)$$

for any arbitrary angle α; therefore the coefficients of the trigonometric functions have to vanish simultaneously. Consequently, the involved scattering coefficients corresponding to the correlations between copolarized and cross-polarized elements in the scattering matrix are subject to the following constraints:

$$\mathrm{Re}\ \sigma_{hhhv} = \mathrm{Re}\ \sigma_{vhvv} = 0 \quad \text{and} \quad \mathrm{Re}\ \sigma_{hhvh} = \mathrm{Re}\ \sigma_{hvvv} \qquad (14a)$$

$$\mathrm{Im}\ \sigma_{hhhv} = \mathrm{Im}\ \sigma_{vhvv} = 0 \qquad (14b)$$

Similarly, transmitting v and receiving v and h in the linear polarization basis rotated by $\alpha + (\pi/2)$ provide the same values for the scattering coefficients of the medium with the reflection symmetry as measured by transmitting h and receiving h and v in the basis rotated by $-\alpha$. Described mathematically, this condition is

$$\sigma'_{vvhv}\left(\alpha + \frac{\pi}{2}\right) = \sigma'_{hhvh}(-\alpha) \qquad (15)$$

which requires from (A7) for $\sigma'^*_{hvvv}(\alpha + \pi/2) = \sigma'_{vvhv}(\alpha + \pi/2)$ and (A3) for $\sigma'_{hhvh}(-\alpha)$ that the real and the imaginary parts of the scattering coefficients to satisfy the following constraints:

$$2(\mathrm{Re}\ \sigma_{hvvv} + \mathrm{Re}\ \sigma_{vhvv} - \mathrm{Re}\ \sigma_{hhvh}$$
$$- \mathrm{Re}\ \sigma_{hhhv}) \sin^2 \alpha \cos^2 \alpha + \mathrm{Re}\ \sigma_{hhvh} \cos^2 \alpha$$
$$- \mathrm{Re}\ \sigma_{hvvv} \sin^2 \alpha = 0 \qquad (16a)$$

$$\mathrm{Im}\ \sigma_{hhvh} \cos^2 \alpha + \mathrm{Im}\ \sigma_{hvvv} \sin^2 \alpha = 0 \qquad (16b)$$

Since α is arbitrary, all the coefficients in (16) have to be zero leading to the following conditions:

$$\mathrm{Re}\ \sigma_{hhvh} = \mathrm{Re}\ \sigma_{hvvv} = 0 \qquad \mathrm{Re}\ \sigma_{hhhv} = \mathrm{Re}\ \sigma_{vhvv} \qquad (17a)$$

$$\mathrm{Im}\ \sigma_{hhvh} = \mathrm{Im}\ \sigma_{hvvv} = 0 \qquad (17b)$$

In summary, the reflection symmetry equalizes the scattering coefficients measured in two linear polarization bases with the mirror symmetry about the vertical plane P. This symmetry forces the polarimetric scattering coefficients for the correlations between the copolarized and the cross-polarized elements in the scattering matrix to be nullified. As seen from (14) and (17),

$$\sigma_{hhhv} = \sigma_{hvhh} = \sigma_{vvvh} = \sigma_{vhvv} = 0 \qquad (18a)$$

$$\sigma_{hhvh} = \sigma_{vhhh} = \sigma_{vvhv} = \sigma_{hvvv} = 0 \qquad (18b)$$

Use of other equations in the appendix leads to the same conclusion. The covariance matrix characterizing the polarimetric backscattering of a medium with reflection symmetry becomes

$$\mathbf{C} = \begin{bmatrix} \sigma_{hhhh} & 0 & 0 & \sigma_{hhvv} \\ 0 & \sigma_{hvhv} & \sigma_{hvvh} & 0 \\ 0 & \sigma^*_{hvvh} & \sigma_{vhvh} & 0 \\ \sigma^*_{hhvv} & 0 & 0 & \sigma_{vvvv} \end{bmatrix} \tag{19}$$

which has at most eight independent parameters. Note that (19) holds for both reciprocal and nonreciprocal medium. If the rotationally invariant reciprocity relation is imposed, then $\beta = \xi = 0$ for $e \neq 0$ and the covariance matrix of the reciprocal medium with the reflection symmetry contains at most five independent parameters. This covariance matrix reduces to

$$\mathbf{C} = \begin{bmatrix} \sigma_{hhhh} & 0 & \sigma_{hhvv} \\ 0 & \sigma_{hvhv} & 0 \\ \sigma^*_{hhvv} & 0 & \sigma_{vvvv} \end{bmatrix} = \sigma \begin{bmatrix} 1 & 0 & \rho(\gamma)^{1/2} \\ 0 & e & 0 \\ \rho^*(\gamma)^{1/2} & 0 & \gamma \end{bmatrix} \tag{20}$$

The covariance matrices (19) and (20) with the zero elements are derived based on the reflection symmetry without any reference to scattering mechanisms. This result is thus valid for volume scattering, surface scattering, or volume-surface interactions to all scattering orders or to the total scattering effects no matter how dense the medium or how rough the surface is as long as the scattering configuration has the reflection symmetry.

As shown here, reflection symmetry requires the complete decorrelations between the copolarized and cross-polarized scattering elements. This requirements has to be observed by results from theoretical models for the symmetrical media as a matter of self-consistency. In this sense, models for volume and surface scatterings from a medium with the reflection symmetry are investigated. For the volume scattering model [*Nghiem et al.*, 1990], consider an anisotropic layer of random medium consisting of a host medium with permittivity $\varepsilon_b = (3.2 + i0.002)\varepsilon_0$ and a fractional volume $f_s = 5\%$ of prolate spheroidal scatterers with permittivity $\varepsilon_s = (42.0 + i45.0)\varepsilon_0$. The scatterers are described by an exponential correlation function with correlation lengths $l_{\rho'} = 0.3$ mm and $l_{z'} = 1.0$ mm aligned in a direction parallel to the y-z plane and tilted by an angle $\psi = 20°$ from the vertical axis. The scattering configuration is shown in Figure 4*a*, where the upper half-space is the air with permittivity ε_0, the middle layer is the tilted anisotropic random medium of 2.0-m thick with permittivity tensor $\bar{\bar{\varepsilon}}_{\text{eff}} = \text{diag}\,[\varepsilon_{\text{eff}\rho'}, \varepsilon_{\text{eff}\rho'} = (3.57 + i0.043)\varepsilon_0, \varepsilon_{\text{eff}z'} = (4.47 + i0.524)\varepsilon_0]$ obtained from the strong permittivity fluctuation theory (SFT), and the lower half-space is an isotropic homogeneous medium with permittivity $\varepsilon_2 = (60.0 + i40.0)\varepsilon_0$. Identical permeability μ is assumed for the media. This scattering configuration has reflection symmetry about a vertical plane parallel to the tilted axis of the scatterers. It should be pointed out that the configuration does not coincide with itself after a rotation by 180° about any vertical axis. A 5-GHz wave is incident on the layer random medium in the direction $\bar{k}_{0i}$ determined by incident angle $\theta_{0i} = 45°$ from the vertical axis $\hat{z}$ and azimuthal angle ϕ_{0i} from the horizontal axis $\hat{x}$. The scattering coefficients σ_{hhhv} and σ_{hvvv} are calculated under the first-order distorted Born approximation and plotted in Figure 4*b* as a function of azimuthal angle ϕ_{0i}. The plots show that

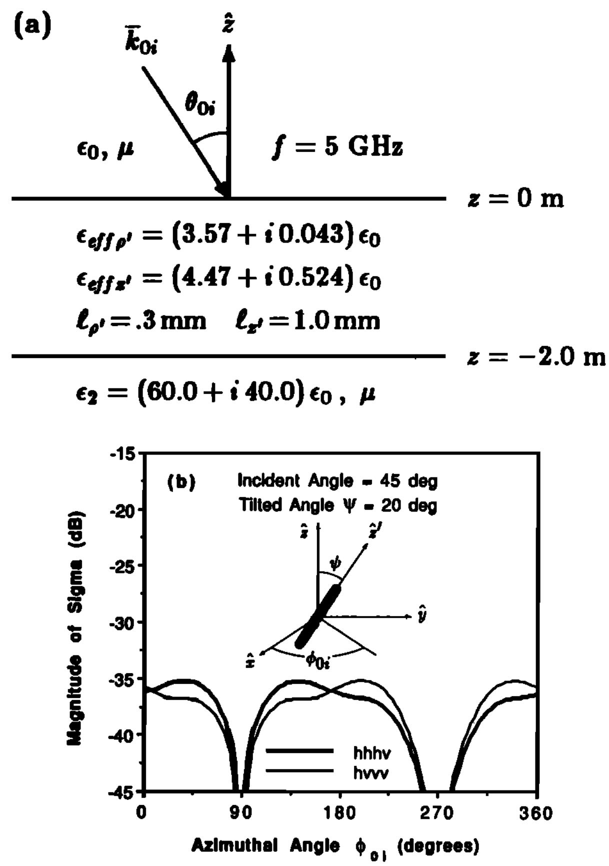

Fig. 4. (*a*) Scattering configuration for a layer of tilted anisotropic random a medium. (*b*) Scattering coefficients σ_{hhhv} and σ_{hvvv} plotted as a function of azimuthal angle ϕ_{0i}.

σ_{hhhv} and σ_{hvvv} have reflection symmetry about $\phi_{0i} = 90°$ and $\phi_{0i} = 270°$, while their values are different as ϕ_{0i} is rotated by 180°. When ϕ_{0i} goes to 90° or 270°, where the scattering configuration has the reflection symmetry, σ_{hhhv} and σ_{hvvv} become zero as required by the symmetry.

For the surface model the scattering from a randomly perturbed periodic surface is formulated by the extended boundary condition method and solved by the perturbation method [*Yueh et al.*, 1989]. The configuration is shown in Figure 5*a* where the large-scale periodic surface is a one-dimensional sinusoidal surface and the random perturbation is a two-dimensional Gaussian random process χ described by a Gaussian correlation function with a correlation length l and a standard deviation σ. This scattering configuration has reflection symmetry about a vertical plane parallel or perpendicular to the row direction and also coincides with itself after a rotation by 180° around a vertical axis. The conventional backscattering coefficients σ_{hh}, σ_{vv}, and σ_{hv} versus the azimuthal angle ϕ_i for a fixed incident angle $\theta_i = 45°$ are graphed in Figure 5*b*, which reveals the reflection symmetry and the periodicity of 180° in ϕ_i as expected. The kinks observed on the σ_{hvhv} curve are due to the relatively abrupt variation of energy distribution between surface and propagating Floquet modes. Figure 5*c* illustrates the results for the magnitudes of scattering coefficients σ_{hhhv} and σ_{hvvv} which are zero when $\phi_i = 0°$, 90°, 180°, and 270°, corresponding to the symmetry directions of the surface.

The models for the volume scattering and the surface scattering with the symmetrical configurations are thus shown to follow the constraints imposed by the reflection symmetry. In geophysical media this symmetry can be observed on water surfaces in the upwind or downwind direction, plowed fields in the direction perpendicular to the row structure, and isotropic and anisotropic scattering media such as forest, snow, or sea ice.

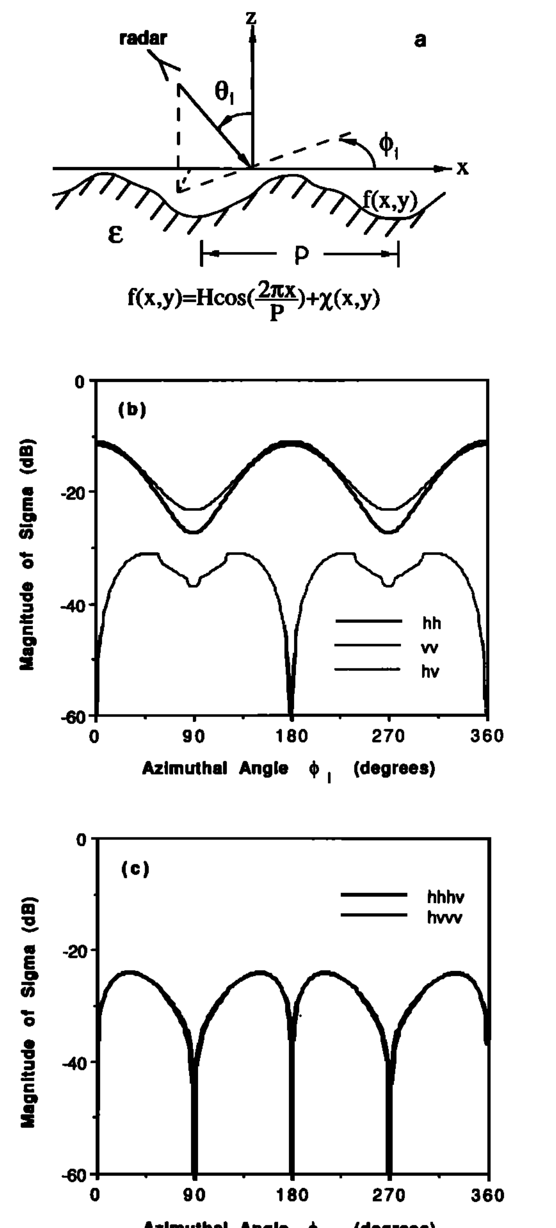

Fig. 5. (*a*) Configuration for a randomly perturbed periodic surface. (*b*) Conventional backscattering coefficients σ_{hh}, σ_{vv}, and σ_{hv} plotted as a function of azimuthal angle ϕ_i at $\theta_i = 45°$ for $\varepsilon = 6.0 + i0.6$, $P = 100$ cm, $H = 10$ cm, correlation length $l = 10$ cm, standard deviation $\sigma = 1$ cm, and frequency $f = 1.4$ GHz. (*c*) Plots of σ_{hhhv} and σ_{hvvv} versus azimuthal angle ϕ_i with the same physical parameters.

4. ROTATION SYMMETRY

In this section the pure rotation symmetry in the two-dimensional linear polarization basis is investigated for the characteristics of the scattering coefficients from a medium with such symmetry. In this group the transformations are labeled by the continuous rotation angle α and denoted by $\mathscr{C}_\infty$ according to group theory notation [*Hamermesh*, 1972]. For a medium with the rotation symmetry the covariance matrix is invariant under the rotation about an axis L by the angle α as illustrated in

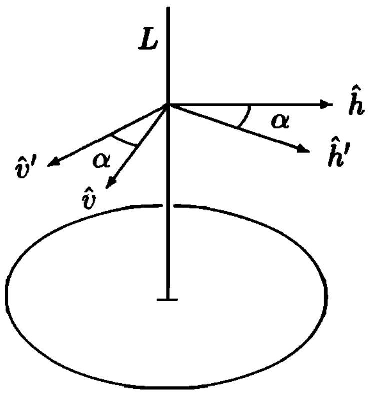

Fig. 6. Rotation symmetry about axis L where the linear polarization basis is $(\hat{h}, \hat{v})$ with $\hat{h} \perp L$ and $\hat{v} \perp L$.

Figure 6. This invariance is used to derive the relations among the polarimetric backscattering coefficients of the medium restricted by the rotation symmetry.

For a wave propagating in parallel to the axis of symmetry (L), both the horizontal $\hat{h}$ and the vertical $\hat{v}$ polarization vectors are perpendicular to axis L. After a rotation by an angle α the covariance matrix remains unchanged; therefore $\sigma_{\mu\nu\tau\kappa} = \sigma'_{\mu\nu\tau\kappa}(\alpha)$. From $\sigma_{hhhh} = \sigma'_{hhhh}$ and (A1) in the appendix, the following results

$$(\sigma_{vvvv} - \sigma_{hhhh}) \sin^2 \alpha + (\sigma_{hvhv} + \sigma_{vhvh} + 2\sigma_{hvvh} - \sigma_{hhhh} - \sigma_{vvvv} + 2 \operatorname{Re} \sigma_{hhvv}) \cos^2 \alpha \sin^2 \alpha + (\operatorname{Re} \sigma_{hhhv} + \operatorname{Re} \sigma_{hhvh}) 2 \cos^3 \alpha \sin \alpha + (\operatorname{Re} \sigma_{hvvv} + \operatorname{Re} \sigma_{vhvv}) 2 \cos \alpha \sin^3 \alpha = 0 \tag{21}$$

hold for any arbitrary angle α. To satisfy (21), the quantities in the parentheses have to vanish simultaneously. The involved scattering coefficients are thus related by

$$\sigma_{hhhh} = \sigma_{vvvv} \tag{22a}$$

$$\sigma_{hvhv} + \sigma_{vhvh} + 2 \operatorname{Re} \sigma_{hvvh} = \sigma_{hhhh} + \sigma_{vvvv} - 2 \operatorname{Re} \sigma_{hhvv} \tag{22b}$$

$$\operatorname{Re} \sigma_{hhhv} + \operatorname{Re} \sigma_{hhvh} = \operatorname{Re} \sigma_{hvvv} + \operatorname{Re} \sigma_{vhvv} = 0 \tag{22c}$$

Similarly, σ_{hhhv} is equal to $\sigma'_{hhhv}(\alpha)$ given by (A2) in the appendix. Taking the real part of the resultant equation gives

$$(\sigma_{hvhv} + \sigma_{vhvh} + 2 \operatorname{Re} \sigma_{hvvh} - \sigma_{hhhh} - \sigma_{vvvv} + 2 \operatorname{Re} \sigma_{hhvv})(\cos^3 \alpha \sin \alpha - \cos \alpha \sin^3 \alpha) - (\operatorname{Re} \sigma_{hhhv} + \operatorname{Re} \sigma_{hhvh} - \operatorname{Re} \sigma_{hvvv} - \operatorname{Re} \sigma_{vhvv}) 4 \cdot \cos^2 \alpha \sin^2 \alpha - (\sigma_{hhhh} - \sigma_{vvvv} - \sigma_{hvhv} + \sigma_{vhvh}) \cdot \cos \alpha \sin \alpha - (\operatorname{Re} \sigma_{hhhv} + \operatorname{Re} \sigma_{vhvv}) 2 \sin^2 \alpha = 0 \tag{23}$$

which introduces the following relations, in addition to (22b) and (22c), on the scattering coefficients

$$\sigma_{hhhh} - \sigma_{vvvv} = \sigma_{hvhv} - \sigma_{vhvh} \tag{24a}$$

$$\operatorname{Re} \sigma_{hhhv} = -\operatorname{Re} \sigma_{vhvv} \tag{24b}$$

Furthermore, equating the imaginary part of σ_{hhhv} to that of $\sigma'_{hhhv}(\alpha)$ yields

$$(\operatorname{Im} \sigma_{hhvv} - \operatorname{Im} \sigma_{hvvh}) \cos \alpha \sin \alpha - (\operatorname{Im} \sigma_{hhhv} - \operatorname{Im} \sigma_{vhvv}) \sin^2 \alpha = 0 \tag{25}$$

in which the coefficients of the trigonometric functions have to be zero for (25) to hold true for an arbitiary α. Consequently,

$$\operatorname{Im} \sigma_{hhvv} = \operatorname{Im} \sigma_{hvvh} \tag{26a}$$

$$\operatorname{Im} \sigma_{hhhv} = \operatorname{Im} \sigma_{vhvv} \tag{26b}$$

Next, using $\sigma_{hhvh} = \sigma'_{hhvh}(\alpha)$ with (A3) from the appendix and examining the real part and the imaginary part lead to

$$(\sigma_{hvhv} + \sigma_{vhvh} + 2 \operatorname{Re} \sigma_{hvvh} - \sigma_{hhhh} - \sigma_{vvvv} + 2 \operatorname{Re} \sigma_{hhvv})(\cos^3 \alpha \sin \alpha - \cos \alpha \sin^3 \alpha) - (\operatorname{Re} \sigma_{hhhv} + \operatorname{Re} \sigma_{hhvh} - \operatorname{Re} \sigma_{hvvv} - \operatorname{Re} \sigma_{vhvv}) 4 \cdot \cos^2 \alpha \sin^2 \alpha - (\sigma_{hhhh} - \sigma_{vvvv} + \sigma_{hvhv} - \sigma_{vhvh}) \cdot \cos \alpha \sin \alpha - (\operatorname{Re} \sigma_{hhvh} + \operatorname{Re} \sigma_{hvvv}) 2 \sin^2 \alpha = 0 \tag{27a}$$

$$(\operatorname{Im} \sigma_{hhvv} + \operatorname{Im} \sigma_{hvvh}) \cos \alpha \sin \alpha - (\operatorname{Im} \sigma_{hhvh} - \operatorname{Im} \sigma_{hvvv}) \sin^2 \alpha = 0 \tag{27b}$$

respectively, which further impose the following conditions on the scattering coefficients:

$$\sigma_{hhhh} - \sigma_{vvvv} = \sigma_{vhvh} - \sigma_{hvhv} \tag{28a}$$

$$\text{Re } \sigma_{hhvh} = -\text{Re } \sigma_{hvvv} \tag{28b}$$

$$\text{Im } \sigma_{hhvv} = -\text{Im } \sigma_{hvvh} \tag{29a}$$

$$\text{Im } \sigma_{hhvh} = \text{Im } \sigma_{hvvv} \tag{29b}$$

Also, inspecting the real part of the relation $\sigma_{hhvv} = \sigma'_{hhvv}(\alpha)$ for the scattering coefficient correlating the two diagonal elements in the scattering matrix results in

$$\begin{aligned}(\sigma_{hvhv} + \sigma_{vhvh} &+ 2 \text{ Re } \sigma_{hvvh} - \sigma_{hhhh} - \sigma_{vvvv} \\ &+ 2 \text{ Re } \sigma_{hhvv}) \cos^2 \alpha \sin^2 \alpha \\ &- (\text{Re } \sigma_{hhhv} + \text{Re } \sigma_{hhvh} - \text{Re } \sigma_{hvvv} - \text{Re } \sigma_{vhvv}) \\ &\cdot (\cos \alpha \sin^3 \alpha - \cos^3 \alpha \sin \alpha) = 0 \end{aligned} \tag{30a}$$

where (A4) in the appendix has been used. No new information is obtained from (30a); however, the imaginary part gives

$$\begin{aligned}\text{Im } \sigma_{hhvv} 2 \sin^2 \alpha &+ (\text{Im } \sigma_{hhhv} + \text{Im } \sigma_{hhvh} \\ &- \text{Im } \sigma_{hvvv} - \text{Im } \sigma_{vhvv}) \cos \alpha \sin \alpha = 0 \end{aligned} \tag{30b}$$

which requires that the relevant scattering coefficients satisfy the following conditions:

$$\text{Im } \sigma_{hhvv} = 0 \tag{31a}$$

$$\text{Im } \sigma_{hhhv} + \text{Im } \sigma_{hhvh} = \text{Im } \sigma_{hvvv} + \text{Im } \sigma_{vhvv} \tag{31b}$$

Finally, the relation $\sigma_{hvhv} = \sigma'_{hvhv}$ for the cross-polarized return is used with (A5) from the appendix to arrive at

$$\begin{aligned}(\sigma_{hvhv} + \sigma_{vhvh} &+ 2 \text{ Re } \sigma_{hvvh} - \sigma_{hhhh} - \sigma_{vvvv} \\ &+ 2 \text{ Re } \sigma_{hhvv}) \cos^2 \alpha \sin^2 \alpha \\ &+ (\text{Re } \sigma_{hhhv} - \text{Re } \sigma_{hvvv}) 2 \cos^3 \alpha \sin \alpha \\ &- (\text{Re } \sigma_{hhvh} - \text{Re } \sigma_{vhvv}) 2 \cos \alpha \sin^3 \alpha \\ &+ (\sigma_{hvhv} - \sigma_{vhvh}) \sin^2 \alpha = 0 \end{aligned} \tag{32}$$

which equates the cross-polarized returns σ_{hvhv} and σ_{vhvh} and also the real parts of the off-diagonal scattering coefficients corresponding to the correlations between the copolarized and the cross-polarized elements in the scattering matrix as described by

$$\sigma_{hvhv} = \sigma_{vhvh} \tag{33a}$$

$$\text{Re } \sigma_{hhhv} = \text{Re } \sigma_{hvvv} \tag{33b}$$

$$\text{Re } \sigma_{hhvh} = \text{Re } \sigma_{vhvv} \tag{33c}$$

As derived, the rotation symmetry enforces the conditions in (22), (24), (26), (28), (29), (31), and (33) on the polarimetric backscattering coefficients due to their invariance under the rotation of the linear polarization basis. The results have been obtained by using (A1)–(A5) from the appendix for the rotated scattering coefficients; the same conclusions are reached by use of other equations in the appendix. In summary, the scattering coefficients of a medium with rotation symmetry possess the following characteristics:

$$\text{Re } \sigma_{hhhv} = -\text{Re } \sigma_{vhvv} = \text{Re } \sigma_{hvvv} \tag{34a}$$

$$\text{Im } \sigma_{hhhv} = \text{Im } \sigma_{vhvv} \tag{34b}$$

$$\text{Re } \sigma_{hhvh} = -\text{Re } \sigma_{hvvv} = \text{Re } \sigma_{vhvv} \tag{35a}$$

$$\text{Im } \sigma_{hhvh} = \text{Im } \sigma_{hvvv} \tag{35b}$$

$$\text{Im } \sigma_{hhvv} = 0 \tag{36a}$$

$$\text{Im } \sigma_{hvvh} = 0 \tag{36b}$$

$$\sigma_{hhhh} = \sigma_{vvvv} \tag{37a}$$

$$\sigma_{hvhv} = \sigma_{vhvh} \tag{37b}$$

$$\begin{aligned}\sigma_{hvhv} + \sigma_{vhvh} &+ 2 \text{ Re } \sigma_{hvvh} \\ &= \sigma_{hhhh} + \sigma_{vvvv} - 2 \text{ Re } \sigma_{hhvv} \end{aligned} \tag{38}$$

These conditions reduce the number of independent parameters in the four-by-four covariance matrix for backscattering from 16 to 6 at most. The corresponding covariance matrix can be expressed as

$$\mathbf{C} = \begin{bmatrix} \sigma_{hhhh} & \text{Re } \sigma_{hhhv} + i \text{ Im } \sigma_{hhhv} & -\text{Re } \sigma_{hhhv} + i \text{ Im } \sigma_{hhvh} & \text{Re } \sigma_{hhvv} \\ \text{Re } \sigma_{hhhv} - i \text{ Im } \sigma_{hhhv} & \sigma_{hvhv} & \text{Re } \sigma_{hvvh} & \text{Re } \sigma_{hhhv} + i \text{ Im } \sigma_{hhvh} \\ -\text{Re } \sigma_{hhhv} - i \text{ Im } \sigma_{hhvh} & \text{Re } \sigma_{hvvh} & \sigma_{hvhv} & -\text{Re } \sigma_{hhhv} + i \text{ Im } \sigma_{hhhv} \\ \text{Re } \sigma_{hhvv} & \text{Re } \sigma_{hhhv} - i \text{ Im } \sigma_{hhvh} & -\text{Re } \sigma_{hhhv} - i \text{ Im } \sigma_{hhhv} & \sigma_{hhhh} \end{bmatrix} \tag{39}$$

where σ_{hhhh}, σ_{hvhv}, and Re σ_{hvhv} are related by σ_{hvhv} + Re σ_{hvvh} = σ_{hhhh} − Re σ_{hhvv}. It is also interesting to note that (34), (35), and (38) can be rewritten as

$$\frac{\mathrm{Re}\ \sigma_{hhhv} + \mathrm{Re}\ \sigma_{hhvh}}{2} = \frac{\mathrm{Re}\ \sigma_{hvvv} + \mathrm{Re}\ \sigma_{vhvv}}{2} = 0 \tag{40a}$$

$$\frac{\mathrm{Im}\ \sigma_{hhhv} + \mathrm{Im}\ \sigma_{hhvh}}{2} = \frac{\mathrm{Im}\ \sigma_{hvvv} + \mathrm{Im}\ \sigma_{vhvv}}{2} \tag{40b}$$

$$\frac{\sigma_{hvhv} + \sigma_{vhvh} + 2\ \mathrm{Re}\ \sigma_{hvvh}}{4} = \frac{\sigma_{hhhh} + \sigma_{vvvv} - 2\ \mathrm{Re}\ \sigma_{hhvv}}{4} \tag{40c}$$

Equations (40*a*) and (40*b*) relate the arithmetic average of the real parts and the imaginary parts of the cross-scattering coefficients, respectively. Moreover, (40*c*) can be expressed simply as $\langle|(f_{hv} + f_{vh})/2)|^2\rangle = \langle|(f_{hh} - f_{vv})/2|^2\rangle$, indicating that the average cross-polarized return just corresponds to the correlation of the difference between the copolarized scattering elements for a medium with the rotation symmetry. Since the reciprocal relation has not been imposed on the derivation in this section so far, the results in (34)–(38) are applicable to a nonreciprocal medium. Rotation symmetry and nonreciprocity are observed in a gyrotropic random medium such as the Earth's ionosphere, which is a plasma magnetized by the geomagnetic field, containing rodlike density irregularities along the field-aligned direction.

When the medium is reciprocal, the reciprocity relation $f_{hv} = f_{vh}$ applies. With the use of the reciprocity relation, (40*c*) becomes

$$\sigma_{hvhv} = (\sigma_{hhhh} - \sigma_{hhvv})/2 \tag{41a}$$

or

$$e = (1 - \rho)/2 \tag{41b}$$

where σ_{hhvv} and ρ are real, implicitly. The covariance matrix for a reciprocal medium with the rotation symmetry takes on the simple form of

$$\mathbf{C} = \begin{bmatrix} \sigma_{hhhh} & i\ \mathrm{Im}\ \sigma_{hhhv} & \mathrm{Re}\ \sigma_{hhvv} \\ -i\ \mathrm{Im}\ \sigma_{hhhv} & \sigma_{hvhv} & i\ \mathrm{Im}\ \sigma_{hhhv} \\ \mathrm{Re}\ \sigma_{hhvv} & -i\ \mathrm{Im}\ \sigma_{hhhv} & \sigma_{hhhh} \end{bmatrix} \tag{42a}$$

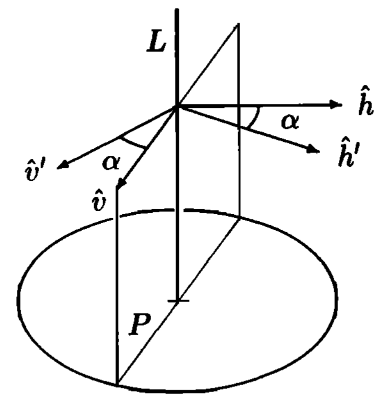

Fig. 7. Azimuthal symmetry obtained from the rotation symmetry by adjoining the reflection symmetry in any vertical plane (P) containing the axis of rotation symmetry (L) where the linear polarization basis is ($\hat{h}$, $\hat{v}$) with $\hat{h} \perp L$ and $P \parallel \hat{v} \perp L$.

$$\mathbf{C} = \sigma \begin{bmatrix} 1 & i\ \mathrm{Im}\ \beta(e)^{1/2} & \mathrm{Re}\ \rho \\ -i\ \mathrm{Im}\ \beta(e)^{1/2} & e & i\ \mathrm{Im}\ \beta(e)^{1/2} \\ \mathrm{Re}\ \rho & -i\ \mathrm{Im}\ \beta(e)^{1/2} & 1 \end{bmatrix} \tag{42b}$$

In this case the covariance matrix contains at most three independent parameters. A chiral medium made by embedding helixes in an isotropic background can be considered as having both reciprocity and rotation symmetry.

5. AZIMUTHAL SYMMETRY

Azimuthal symmetry is studied in this section to reveal its effects on the polarimetric backscattering coefficients. The azimuthal symmetry group, denoted by $\mathscr{C}_{\infty v}$, can be obtained from the rotation group $\mathscr{C}_\infty$ by adjoining the reflection σ_v in any vertical plane (P) passing through the axis of rotation symmetry (L) as depicted in Figure 7, where the polarization basis is ($\hat{h}$, $\hat{v}$) with $\hat{h} \perp L$ and $P \parallel \hat{v} \perp L$. Thus azimuthal symmetry has the characteristics of both the reflection and the rotation symmetries. The combination of the results from the two symmetries, discussed in sections 3 and 4, provides a set of equations relating the scattering coefficients of a medium with azimuthal symmetry. These equations will be used to test the calculations for the volume scattering from layer random media under the first-order distorted Born approximation and for the rough surface scattering under the first-order small perturbation method.

As indicated, a configuration with the azimuthal symmetry also has reflection symmetry. The scattering coefficients σ_{hhhv}, σ_{hhvh}, σ_{hvhh}, σ_{hvvv}, σ_{vhhh}, σ_{vhvv}, σ_{vvhv}, and σ_{vvvh} are therefore zero in all azimuthal directions of observation. Moreover, azimuthal symmetry imposes further constraints on the scattering coefficients due to the rotation symmetry as specified in (36), (37), and (38). The covariance matrix, which has at most three independent parameters, of a scattering medium with the azimuthal symmetry can thus be written as

$$\mathbf{C} = \begin{bmatrix} \sigma_{hhhh} & 0 & 0 & \mathrm{Re}\,\sigma_{hhvv} \\ 0 & \sigma_{hvhv} & \sigma_{hhhh} - \sigma_{hvhv} - \mathrm{Re}\,\sigma_{hhvv} & 0 \\ 0 & \sigma_{hhhh} - \sigma_{hvhv} - \mathrm{Re}\,\sigma_{hhvv} & \sigma_{hvhv} & 0 \\ \mathrm{Re}\,\sigma_{hhvv} & 0 & 0 & \sigma_{hhhh} \end{bmatrix} \tag{43a}$$

$$\mathbf{C} = \begin{bmatrix} \sigma_{hhhh} & 0 & 0 & \sigma_{hhhh} - \sigma_{hvhv} - \mathrm{Re}\,\sigma_{hvvh} \\ 0 & \sigma_{hvhv} & \mathrm{Re}\,\sigma_{hvvh} & 0 \\ 0 & \mathrm{Re}\,\sigma_{hvvh} & \sigma_{hvhv} & 0 \\ \sigma_{hhhh} - \sigma_{hvhv} - \mathrm{Re}\,\sigma_{hvvh} & 0 & 0 & \sigma_{hhhh} \end{bmatrix} \tag{43b}$$

If a medium with the azimuthal symmetry is also reciprocal, $\sigma_{hhhv} = \sigma_{hvvv} = 0$ or $\beta = \xi = 0$, and the nonzero scattering coefficients are constrained by

$$\sigma_{hhhh} = \sigma_{vvvv}, \qquad \mathrm{Im}\,\sigma_{hhvv} = 0, \qquad \sigma_{hvhv} = (\sigma_{hhhh} + \sigma_{vvvv} - 2\,\mathrm{Re}\,\sigma_{hhvv})/4 \tag{44a}$$

or

$$\gamma = 1, \qquad \mathrm{Im}\,\rho = 0, \qquad e = (1 + \gamma - 2\,\mathrm{Re}\,\rho)/4 \tag{44b}$$

where (44b) is the normalized form of (44a). Note that $\sigma_{hhhv} = \sigma_{hvvv} = 0$ or $\beta = \xi = 0$ is not the necessary condition of (44) which is derived independently from a different symmetry group. The three-by-three covariance matrix (9) of the reciprocal medium with the azimuthal symmetry has at most two independent parameters and can be written simply as

$$\mathbf{C} = \begin{bmatrix} \sigma_{hhhh} & 0 & \sigma_{hhvv} \\ 0 & (\sigma_{hhhh} - \sigma_{hhvv})/2 & 0 \\ \sigma_{hhvv} & 0 & \sigma_{hhhh} \end{bmatrix} = \begin{bmatrix} \sigma_{hhhh} & 0 & \sigma_{hhhh} - 2\sigma_{hvhv} \\ 0 & \sigma_{hvhv} & 0 \\ \sigma_{hhhh} - 2\sigma_{hvhv} & 0 & \sigma_{hhhh} \end{bmatrix} \tag{45a}$$

$$\mathbf{C} = \sigma \begin{bmatrix} 1 & 0 & \rho \\ 0 & (1-\rho)/2 & 0 \\ \rho & 0 & 1 \end{bmatrix} = \sigma \begin{bmatrix} 1 & 0 & 1-2e \\ 0 & e & 0 \\ 1-2e & 0 & 1 \end{bmatrix} \tag{45b}$$

The equations for the scattering coefficients hold for all scattering mechanisms in the azimuthally symmetrical medium. Results calculated from theoretical models have to be consistent with the above conditions imposed on the scattering coefficients due to azimuthal symmetry. As a verification, volume scattering from layer random medium and rough surface scattering from a medium interface are considered here. For volume scattering, the wave theory is used to construct models [*Nghiem*, 1991] for layer inhomogeneous media, and the backscattering coefficients are calculated under the first-order distorted Born approximation with effective permittivities derived from the strong fluctuation theory. Figure 8a is the scattering configuration similar to the volume scattering case in section 3 except that the anisotropic scattering layer includes ellipsoidal scatterers. These scatterers are preferentially oriented in the vertical direction and randomly oriented in the azimuthal directions. A three-dimensional correlation function with correlation lengths $l_{x'} = 0.1$ mm, $l_{y'} = 0.3$ mm, and $l_{z'} = 1.0$ mm is used to describe the ellipsoidal scatterers. The calculations show that $\beta = \xi = 0$ for all incident and azimuthal angles as required by the symmetry. For incident angles up to 65°, Figure 8b compares the ratio e calculated directly from the model with those calculated from

$$e_1 = (1 - |\rho|)/2 \tag{46a}$$

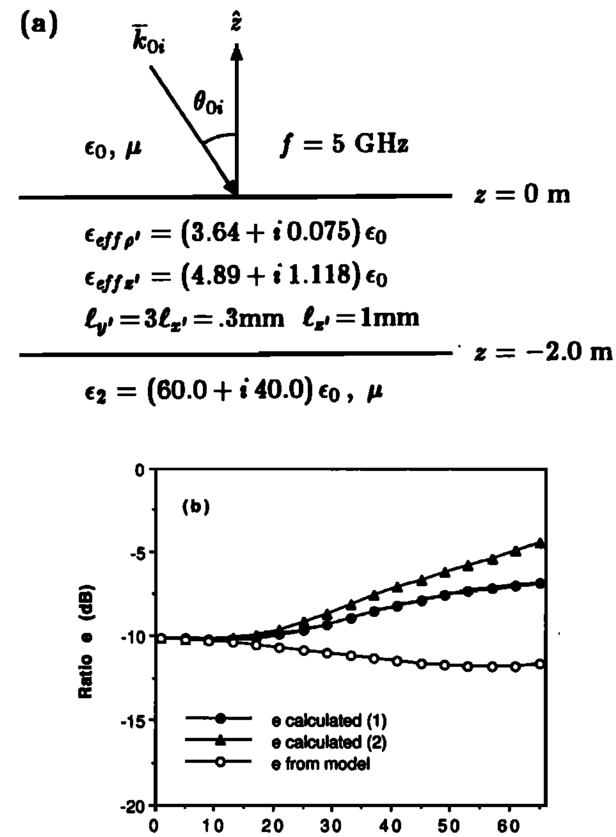

Fig. 8. (*a*) Scattering configuration for a layer of vertical ellipsoids with random azimuthal orientation. (*b*) Comparison of the ratio e calculated directly from the model with e_1 (1) and e_2 (2) calculated from (46) as a function of incident angle θ_{0i}.

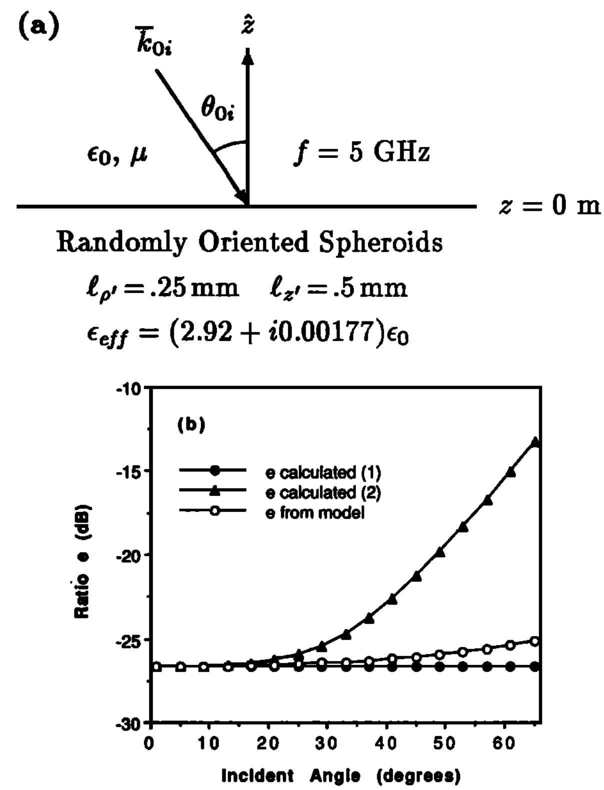

Fig. 9. (*a*) Scattering configuration for a half-space of a medium containing randomly oriented prolate spheroidal scatterers. (*b*) Comparison of the ratio e calculated directly from the model with e_1 (1) and e_2 (2) calculated from (46) as a function of incident angle θ_{0i}.

$$e_2 = [1 + \gamma - 2 \operatorname{Re} \rho(\gamma)^{1/2}]/4 \qquad (46b)$$

based on ρ and γ from the model. It is obvious that e_2 reduces to e_1 with $\gamma = 1$ and Im $\rho = 0$. At normal incidence, where the configuration has the azimuthal symmetry, the results exactly coincide; that is, $e = e_1 = e_2$. As the incident angle increases, the observation direction turns away from the axis of symmetry, and the results deviate since the layer boundary and the vertical orientation of the scatterers are not azimuthally symmetrical at oblique incident angles. In this case, $\sigma_{hhhv} = \sigma_{hvvv} = 0$ are still valid since the condition is the reflection symmetry and the azimuthal symmetry is thus an overrequirement.

Consider another case of scattering configuration in Figure 9*a*, where the lower medium contains randomly oriented spheroidal scatterers [*Nghiem*, 1991]. The host medium has a permittivity of $\varepsilon_b = (3.2 + i0.002)\varepsilon_0$, and the scatterer permittivity is $\varepsilon_s = \varepsilon_0$. The scatterers are described by an exponential correlation function of spheroidal form with correlation lengths $l_{\rho'} = 0.25$ mm and $l_{z'} = 0.5$ mm. The polarimetric scattering coefficients at 5 GHz are also calculated under the first-order distorted Born approximation with an isotropic effective permittivity $\varepsilon_{\text{eff}} = (2.92 + i1.77 \times 10^{-3})\varepsilon_0$ obtained from SFT for 10% fractional volume of the scatterers. The results for ratio e are plotted in Figure 9*b*, which also shows that e ratios are the same at normal incidence due to the azimuthal symmetry. Similar to the last case, the new results are different at larger incident angles since the azimuthal symmetry is destroyed. While e_1 still stays away from e, e_1 is closer to e compared to the case of the configuration in Figure 8. The reason is that the azimuthal asymmetry of the layer-embedding randomly oriented spheroidal scatterers is only caused by the boundary effects.

For a slightly rough surface the backscatters for h and v polarizations are completely correlated under the first-order small perturbation approximation. In this case the covariance matrix for the rough interface has the form

$$\mathbf{C} = \sigma \begin{bmatrix} 1 & 0 & (\gamma)^{1/2} \\ 0 & 0 & 0 \\ (\gamma)^{1/2} & 0 & \gamma \end{bmatrix} \tag{47}$$

where $\sigma = \sigma_{hh}$, $\gamma = \sigma_{vv}/\sigma_{hh}$, and the analytical solutions for σ_{hh} and σ_{vv} are given by *Tsang et al.* [1985]. At normal incidence, $\gamma = 1$, and the covariance matrix (47) of the scattering from the rough surface trivially satisfies (45*b*) due to the azimuthal symmetry. At larger incident angles, $\gamma \neq 1$, and (47) is different from (45*b*) since the scattering configuration departs from the symmetry. As shown, the theoretical models for the volume and the surface scatterings under consideration consistently follow the conditions imposed by the azimuthal symmetry on the backscattering coefficients.

6. CENTRICAL SYMMETRY

This section discusses the polarimetric backscattering coefficients from scatterers or media with centrical symmetry about a point. Scattering from leaves with random orientation can be considered as an example of this symmetry. Centrical symmetry also has reflection symmetry about any plane containing the point of symmetry and azimuthal symmetry about any axis passing through the symmetry point. Thus centrical symmetry can be considered as azimuthal symmetry with the axis containing the center point and rotated in three dimensions. Let the linear polarization basis ($\hat{h}$, $\hat{v}$) be perpendicular to the observation direction toward the point of the centrical symmetry O as depicted in Figure 10. In this polarization basis, all symmetry conditions derived in this paper are therefore valid at all azimuthal and incident angles. These conditions will be used to verify the symmetry consistency in volume and surface scattering models.

For volume scattering the backscattering coefficients from a layer of randomly oriented scatterers with spheroidal shapes are studied. The scattering configuration is illustrated in Figure 11*a*, where the upper half-space is air, the middle layer is composed of an air background with the spheroidal

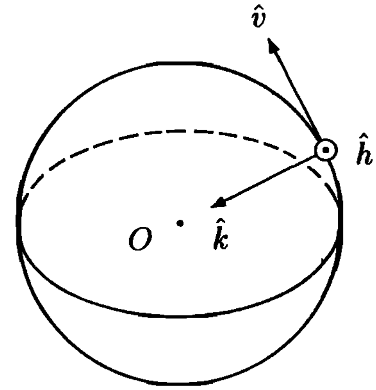

Fig. 10. Centrical symmetry about point O to which the unit incident wave vector $\hat{k}$ of the linear polarization basis ($\hat{h}$, $\hat{v}$) is directed.

scatterers, and the underlying medium is homogeneous. The diffuse boundary condition between the air and the scattering layer is represented by the dashed line in Figure 11*a*. The model has been developed for applications in remote sensing of vegetation canopies [*Nghiem et al.*, 1991]. The frequency under consideration is 5 GHz for a wave incident at a variable angle θ_{0i}. The scatterers have permittivity $\varepsilon_s = (30.0 + i10.0)\varepsilon_0$ and occupy a fractional volume of 0.5%. The underlying medium has a homogeneous permittivity of $\varepsilon_2 = (9.0 + i1.5)\varepsilon_0$. The thickness of the scattering layer is taken to be 5 m so that the wave is highly attenuated before the lower interface is reached. The polarimetric scattering coefficients are calculated from the model under the first-order Born approximation. Figure 11*b* shows the conventional backscattering coefficients σ_{hh}, σ_{vv}, and σ_{hv} (where $\sigma_{hh} = \sigma_{vv}$), which drop more than 4 dB over the range of incident angles. Figure 11*c* graphs the results for the correlation coefficient ρ which is insensitive to the incident angles. Figure 11*d* reports the results for the e ratios obtained from the model and from the calculations with (46). Rather, corresponding to the behavior of ρ, the e ratios remain unchanged as the incident angles increases. Moreover, the results coincide at all incident angles as observed from the plots. This is true for oblique incidence because the scatterers are randomly oriented and the boundary effects are insignificant for the diffuse upper interface and the unreachable lower surface. The centrical symmetry is therefore preserved virtually for all incident direction from the upper half-space. It

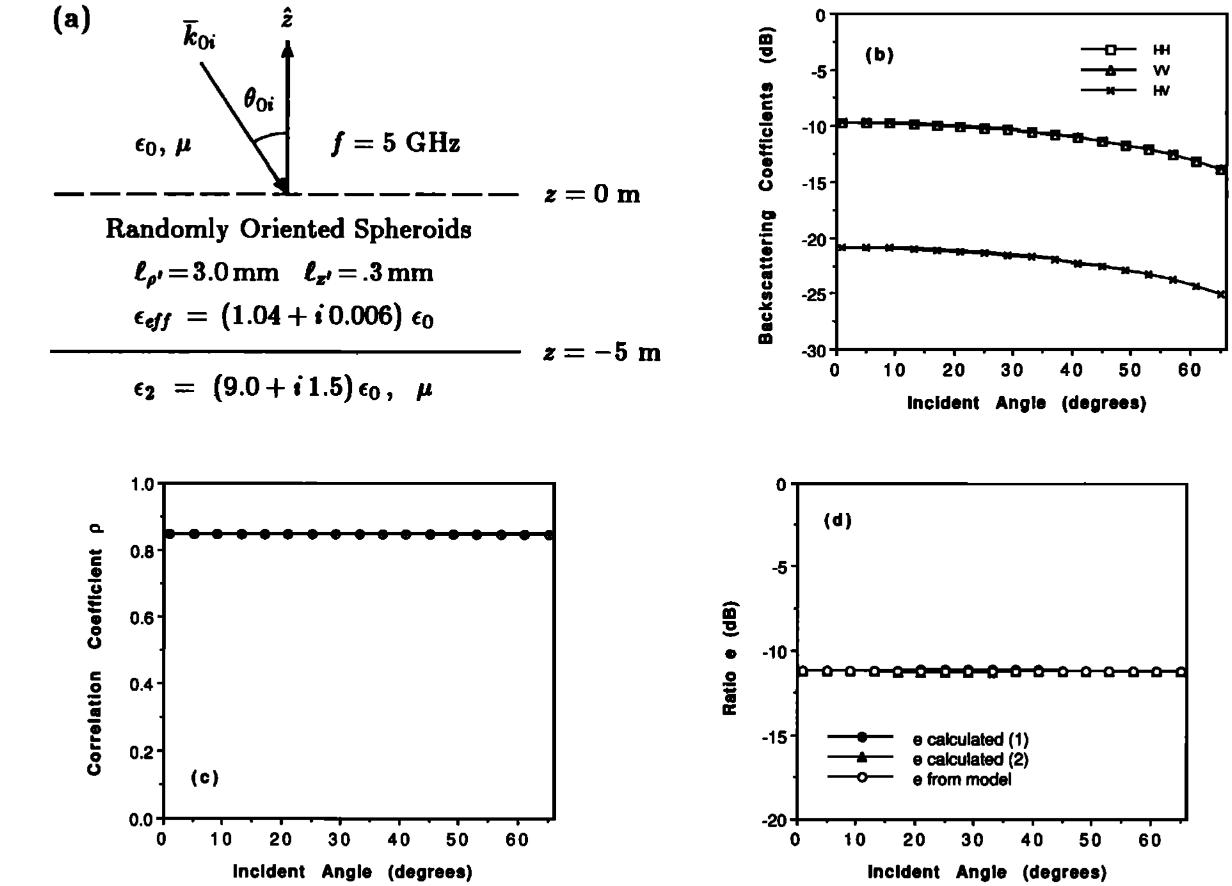

Fig. 11. (*a*) Scattering configuration for a layer of randomly oriented prolate spheroidal scatterers; the dashed line represents the diffuse boundary between the air and the scattering layer. (*b*) Conventional backscattering coefficients σ_{hh}, σ_{vv}, and σ_{hv}. (*c*) Correlation coefficient ρ. (*d*) Results for e ratios obtained from model and from (46).

should be noted that the main difference from the configuration in Figure 9 is the boundary condition at the interface between the upper half-space and the scattering medium. Also, from the model the results for β and ξ are zero. Thus the backscattering from the randomly oriented scatterers with diffuse boundary follows the conditions in (44) at arbitrary incident angles due to the centrical symmetry.

For backscattering from the rough surface the geometrical optics approximation corresponds to the assumption of specular point return, and the backscattering for h and v polarizations is equal and completely correlated. The covariance matrix describing the scattering from the rough surface under the geometric optics condition becomes

$$\mathbf{C} = \sigma \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} \tag{48}$$

where $\sigma = \sigma_{hh} = \sigma_{vv}$ whose analytical form is given by *Tsang et al.* [1985]. The covariance matrix (48) obeys the symmetry conditions in (44) in a trivial manner. Even though there is a boundary in this case of rough surface scattering, the specular points responsible for the scattering from the surface are centrically symmetrical, as observed at any incident angle under the assumption of geometrical optics. As shown, the scattering coefficients calculated from the theoretical models for the volume and the surface scatterings follow the conditions imposed by the symmetry.

7. APPLICATIONS

7.1. *Medium structures*

The behavior of the polarimetric backscattering coefficients based on the symmetry properties can be used as a reference to study the structure of the scatterers in geophysical media. Different geometrical distribution of nonspherical scatterers can be identified and classified by comparing the scattering coefficients with the corresponding symmetry calculations.

As discussed, water waves on the ocean surface have reflection symmetry with respect to the vertical plane parallel to the wind direction, and the corresponding covariance matrix contains the zero-scattering coefficients as specified by (18). These scattering coefficients become nonzero and vary periodically as the azimuthal angle turns around and return to the zero value again when the azimuthal direction coincides with the wind direction. For sea ice the first-year ice type has a preferential vertical structure observed in the orientation of brine inclusions [*Weeks and Ackley*, 1982], while scatterers in multiyear ice are more randomly distributed. Comparisons of the measured value for the polarimetric returns from sea ice with the symmetry calculations can reveal the structural information which helps identify the ice types. Agricultural plants are usually grown in rows for many species of vegetables. This row structure for vegetation has reflection symmetry about vertical planes perpendicular and parallel to the row direction, where the covariance matrix has the form of (20). Azimuthal symmetry is often observed in forests whose covariance matrix can assume the form of (45). For orientations of vegetation elements the distributions have been formulated to describe various structures including spherical, uniform, planophile, plagiothile, erectophile, and extremophile distributions which can be represented simply by a beta probability density function [*Goel and Strebel*, 1984]. Foliage having leaves with spherical orientation distribution can have centrical symmetry which requires the polarimetric scattering coefficients to follow (44) at arbitrary incident angles. When the frequency is low such that the electromagnetic wave can penetrate through the foliage canopy, the centrical symmetry can be destroyed by the horizontal branches or the vertical tree trunks. In this case the preferential orientations reduce the vegetation medium to azimuthal symmetry at normal incidence and to reflection symmetry at oblique incidence, and the covariance matrix will behave accordingly.

From many experimental campaigns, fully polarimetric data over various geophysical media such as snow, sea ice, vegetation, soil, lava, or ocean have been collected. The available results of an investigation for structural information from extensive data sets obtained by the Jet Propulsion Laboratory synthetic aperture radar are reported by S. V. Nghiem, et al. (Jet Propulsion Laboratory, "Polarimetric remote sensing of geophysical medium structures in geophysical media," unpublished manuscript, 1992). This study provides a selection of distributed targets from which scattering coefficients are well behaved in accordance with the symmetry conditions. For instance, a tropical rain forest with a dense foliage canopy can naturally manifest centrical symmetry.

7.2. *Environmental effects*

Environmental conditions can change the polarimetric signatures from geophysical media through their intervention in the path of the probing wave or through the restructuring of the media themselves. In remote sensing from space the Earth's ionosphere may modify the signatures under some circumstances. Near the equatorial anomalies or during a period of high solar activity, the ionospheric density irregularities can introduce signature distortions which destroy the symmetry behavior of the scattering coefficients from symmetrical media. For example, the covariance matrix of an azimuthally symmetrical medium takes on the form (43) instead of (45) after the field-aligned transionspheric propagation. Thick snow cover on first-year ice can make the polarimetric signature become more isotropic [*Nghiem*, 1991], and the covariance matrix approaches the case of centrical symmetry. Rain can cause anisotropic effects due to the nonspherical shape and the preferential alignment of the rain drops. When it rains over a forest canopy with the centrical symmetry, the covariance matrix in this case can be different from (45). These effects from the environmental conditions can therefore be recognized by inspecting the covariance matrices of the geophysical media having the symmetry properties.

In many instances, the environmental conditions can change the structure of the media. A sea current can align *c* axes in sea ice [*Weeks and Ackley*, 1982] which becomes biaxial, and it has reflection symmetry rather than azimuthal as in sea ice with random orientation of the *c* axes in horizontal directions. Wind bends the canopy on a wheat field and transforms the azimuthal symmetry into reflection symmetry. Leaf inclination also depends on environmental factors such as rain and plant stress.

In some species of grass and agricultural plants (herbaceous vegetation) the leaf inclination varies diurnally [*Le Toan et al.*, 1990], resulting in the variation of the orientation distribution and the corresponding covariance matrix. The azimuthal asymmetry of vegetation can also be attributed to physiological behavior such as heliotropism which may render the reflection symmetry in the heliotropic canopy. Comparison of the covariance matrices of these geophysical media with the polarimetric signatures under normal conditions can provide information pertaining to the environmental effects.

7.3. *Polarimetric calibration*

Correct interpretation of polarimetric data requires calibration of the polarimetric radar, in other words, to estimate the polarization distortion matrices of both transmitting and receiving channels. In this regard, it has been a standard practice to deploy man-made targets such as corner reflectors or active transponders in the scene to be imaged. However, when the man-made targets are not available, the symmetry that exists in many natural distributed targets will be a very useful tool for the polarimetric calibration.

With the applications of symmetry, algorithms are developed to remove the cross talk and channel imbalance in a step-by-step process using the response from natural distributed targets so that the calibration can be carried out as much as possible with the available degree of symmetry (S. H. Yueh, et al., Jet Propulsion Laboratory, "External calibration of polarimetic radars using distributed target," unpublished manuscript, 1992). In general, a polarimetric radar is nonreciprocal and is described by six complex parameters. On the basis of the reciprocity which is usually satisfied for natural distributed targets, the polarimetric radar is made reciprocal by using an equivalent point-target response derived by *Yueh et al.* [1991]. The number of unknown parameters is therefore reduced from six to three, including one for channel imbalance and two for cross talk. When a distributed target with reflection symmetry is available in the imaged scene, the two complex cross-talk parameters are calculated by means of (18). The residual error is the channel imbalance which can be removed by (36*a*) for the phase and (38) for the magnitude if a target with rotation symmetry is also available. It should be noted that these conditions are based only on the knowledge of symmetries and not restricted by types of scattering media or scattering mechanisms. The polarimetric calibration for cross-talk and channel imbalance removals done with the new methods renders a possibility of calibrating a polarimetric radar without the deployment of man-made calibration targets which is usually difficult in harsh environments, such as sea ice and ocean water. Moreover, the new calibration algorithms can be applied to polarimetric remote sensing from space as a result of the large coverage of natural distributed targets such as tropical rain forests.

8. SUMMARY

In this paper the relations among polarimetric backscattering coefficients have been derived from the symmetry properties of the media. The symmetries under consideration are due to reflection, rotation, azimuthal, and centrical symmetry groups. For reflection symmetry the scattering coefficients correlating the copolarized and the cross-polarized elements in the scattering matrix are proved to be zero. For the rotation symmetry the constraints imposed on the scattering coefficients are derived, and the 16 independent parameters in the covariance matrix are reduced to six for nonreciprocal media and to three for reciprocal media. The azimuthal symmetry group is the adjoint between the reflection and the rotation groups giving rise to a set of equations restricting the scattering coefficients of most geophysical media at normal incidence. Centrical symmetry generalizes the characteristics of the azimuthal symmetry to all incident angles including the oblique cases. The derivations in this paper are based only on the symmetry properties of the media, and thus the results are valid for all scattering mechanisms, including volume scattering, surface scattering, and their interactions. Results calculated from theoretical models for media with symmetrical configurations have been shown to obey the symmetry conditions on the scattering coefficients. For volume scattering, models based on the wave theory under the first-order distorted Born approximation for spheroidal and ellipsoidal scatterers with different orientation distributions have been investigated. For surface scattering, randomly perturbed quasi-periodic rough surface and rough interfaces under the first-order small perturbation method and the geometrical optics have been considered. The equations for the scattering coefficients imposed by the symmetries are applicable to gyrotropic, chiral, anisotropic, and isotropic scattering media which encompass all the media encountered in geophysical remote sensing. On the basis of symmetry properties, medium structures for water waves, sea ice, and vegetation and environmental effects, including the ionosphere, snow cover, rain, wind, and sea current are discussed. From the symmetry results in this paper, new methods are proposed for the complete relative calibration of polarimetric radars with only the use of natural distributed targets.

APPENDIX

In this appendix the results are presented for the complete set of scattering coefficients, constituting the full covariance matrix in the rotated linear polarization basis:

$$\begin{aligned}\sigma'_{hhhh} = &+\sigma_{hhhh}\cos^4\alpha + \sigma_{hhhv}\cos^3\alpha\sin\alpha\\ &+\sigma_{hhvh}\cos^3\alpha\sin\alpha + \sigma_{hhvv}\cos^2\alpha\sin^2\alpha\\ &+\sigma_{hvhh}\cos^3\alpha\sin\alpha + \sigma_{hvhv}\cos^2\alpha\sin^2\alpha\\ &+\sigma_{hvvh}\cos^2\alpha\sin^2\alpha + \sigma_{hvvv}\cos\alpha\sin^3\alpha\\ &+\sigma_{vhhh}\cos^3\alpha\sin\alpha + \sigma_{vhhv}\cos^2\alpha\sin^2\alpha\\ &+\sigma_{vhvh}\cos^2\alpha\sin^2\alpha + \sigma_{vhvv}\cos\alpha\sin^3\alpha\\ &+\sigma_{vvhh}\cos^2\alpha\sin^2\alpha + \sigma_{vvhv}\cos\alpha\sin^3\alpha\\ &+\sigma_{vvvh}\cos\alpha\sin^3\alpha + \sigma_{vvvv}\sin^4\alpha\end{aligned} \tag{A1}$$

$$\begin{aligned}\sigma'_{hhhv} = &-\sigma_{hhhh}\cos^3\alpha\sin\alpha + \sigma_{hhhv}\cos^4\alpha\\ &-\sigma_{hhvh}\cos^2\alpha\sin^2\alpha + \sigma_{hhvv}\cos^3\alpha\sin\alpha\\ &-\sigma_{hvhh}\cos^2\alpha\sin^2\alpha + \sigma_{hvhv}\cos^3\alpha\sin\alpha\\ &-\sigma_{hvvh}\cos\alpha\sin^3\alpha + \sigma_{hvvv}\cos^2\alpha\sin^2\alpha\\ &-\sigma_{vhhh}\cos^2\alpha\sin^2\alpha + \sigma_{vhhv}\cos^3\alpha\sin\alpha\\ &-\sigma_{vhvh}\cos\alpha\sin^3\alpha + \sigma_{vhvv}\cos^2\alpha\sin^2\alpha\\ &-\sigma_{vvhh}\cos\alpha\sin^3\alpha + \sigma_{vvhv}\cos^2\alpha\sin^2\alpha\\ &-\sigma_{vvvh}\sin^4\alpha + \sigma_{vvvv}\cos\alpha\sin^3\alpha\end{aligned} \tag{A2}$$

$$\begin{aligned}\sigma'_{hhvh} = &-\sigma_{hhhh}\cos^3\alpha\sin\alpha - \sigma_{hhhv}\cos^2\alpha\sin^2\alpha\\ &+\sigma_{hhvh}\cos^4\alpha + \sigma_{hhvv}\cos^3\alpha\sin\alpha\\ &-\sigma_{hvhh}\cos^2\alpha\sin^2\alpha - \sigma_{hvhv}\cos\alpha\sin^3\alpha\\ &+\sigma_{hvvh}\cos^3\alpha\sin\alpha + \sigma_{hvvv}\cos^2\alpha\sin^2\alpha\\ &-\sigma_{vhhh}\cos^2\alpha\sin^2\alpha - \sigma_{vhhv}\cos\alpha\sin^3\alpha\\ &+\sigma_{vhvh}\cos^3\alpha\sin\alpha + \sigma_{vhvv}\cos^2\alpha\sin^2\alpha\\ &-\sigma_{vvhh}\cos\alpha\sin^3\alpha - \sigma_{vvhv}\sin^4\alpha\\ &+\sigma_{vvvh}\cos^2\alpha\sin^2\alpha + \sigma_{vvvv}\cos\alpha\sin^3\alpha\end{aligned} \tag{A3}$$

$$\begin{aligned}\sigma'_{hhvv} = &+\sigma_{hhhh}\cos^2\alpha\sin^2\alpha - \sigma_{hhhv}\cos^3\alpha\sin\alpha\\ &-\sigma_{hhvh}\cos^3\alpha\sin\alpha + \sigma_{hhvv}\cos^4\alpha\\ &+\sigma_{hvhh}\cos\alpha\sin^3\alpha - \sigma_{hvhv}\cos^2\alpha\sin^2\alpha\\ &-\sigma_{hvvh}\cos^2\alpha\sin^2\alpha + \sigma_{hvvv}\cos^3\alpha\sin\alpha\\ &+\sigma_{vhhh}\cos\alpha\sin^3\alpha - \sigma_{vhhv}\cos^2\alpha\sin^2\alpha\\ &-\sigma_{vhvh}\cos^2\alpha\sin^2\alpha + \sigma_{vhvv}\cos^3\alpha\sin\alpha\\ &+\sigma_{vvhh}\sin^4\alpha - \sigma_{vvhv}\cos\alpha\sin^3\alpha\\ &-\sigma_{vvvh}\cos\alpha\sin^3\alpha + \sigma_{vvvv}\cos^2\alpha\sin^2\alpha\end{aligned} \tag{A4}$$

$$\begin{aligned}\sigma'_{hvhv} = &+\sigma_{hhhh}\cos^2\alpha\sin^2\alpha - \sigma_{hhhv}\cos^3\alpha\sin\alpha\\ &+\sigma_{hhvh}\cos\alpha\sin^3\alpha - \sigma_{hhvv}\cos^2\alpha\sin^2\alpha\\ &-\sigma_{hvhh}\cos^3\alpha\sin\alpha + \sigma_{hvhv}\cos^4\alpha\\ &-\sigma_{hvvh}\cos^2\alpha\sin^2\alpha + \sigma_{hvvv}\cos^3\alpha\sin\alpha\\ &+\sigma_{vhhh}\cos\alpha\sin^3\alpha - \sigma_{vhhv}\cos^2\alpha\sin^2\alpha\\ &+\sigma_{vhvh}\sin^4\alpha - \sigma_{vhvv}\cos\alpha\sin^3\alpha\\ &-\sigma_{vvhh}\cos^2\alpha\sin^2\alpha + \sigma_{vvhv}\cos^3\alpha\sin\alpha\\ &-\sigma_{vvvh}\cos\alpha\sin^3\alpha + \sigma_{vvvv}\cos^2\alpha\sin^2\alpha\end{aligned} \tag{A5}$$

$$\begin{aligned}\sigma'_{hvvh} = &+\sigma_{hhhh}\cos^2\alpha\sin^2\alpha + \sigma_{hhhv}\cos\alpha\sin^3\alpha\\ &-\sigma_{hhvh}\cos^3\alpha\sin\alpha - \sigma_{hhvv}\cos^2\alpha\sin^2\alpha\\ &-\sigma_{hvhh}\cos^3\alpha\sin\alpha - \sigma_{hvhv}\cos^2\alpha\sin^2\alpha\\ &+\sigma_{hvvh}\cos^4\alpha + \sigma_{hvvv}\cos^3\alpha\sin\alpha\\ &+\sigma_{vhhh}\cos\alpha\sin^3\alpha + \sigma_{vhhv}\sin^4\alpha\\ &-\sigma_{vhvh}\cos^2\alpha\sin^2\alpha - \sigma_{vhvv}\cos\alpha\sin^3\alpha\\ &-\sigma_{vvhh}\cos^2\alpha\sin^2\alpha - \sigma_{vvhv}\cos\alpha\sin^3\alpha\\ &+\sigma_{vvvh}\cos^3\alpha\sin\alpha + \sigma_{vvvv}\cos^2\alpha\sin^2\alpha\end{aligned} \tag{A6}$$

$$\begin{aligned}\sigma'_{hvvv} = &-\sigma_{hhhh} \cos\alpha \sin^3\alpha + \sigma_{hhhv} \cos^2\alpha \sin^2\alpha \\ &+ \sigma_{hhvh} \cos^2\alpha \sin^2\alpha - \sigma_{hhvv} \cos^3\alpha \sin\alpha \\ &+ \sigma_{hvhh} \cos^2\alpha \sin^2\alpha - \sigma_{hvhv} \cos^3\alpha \sin\alpha \\ &- \sigma_{hvvh} \cos^3\alpha \sin\alpha + \sigma_{hvvv} \cos^4\alpha - \sigma_{vhhh} \sin^4\alpha \\ &+ \sigma_{vhhv} \cos\alpha \sin^3\alpha + \sigma_{vhvh} \cos\alpha \sin^3\alpha \\ &- \sigma_{vhvv} \cos^2\alpha \sin^2\alpha + \sigma_{vvhh} \cos\alpha \sin^3\alpha \\ &- \sigma_{vvhv} \cos^2\alpha \sin^2\alpha - \sigma_{vvvh} \cos^2\alpha \sin^2\alpha \\ &+ \sigma_{vvvv} \cos^3\alpha \sin\alpha \end{aligned} \tag{A7}$$

$$\begin{aligned}\sigma'_{vhvh} = &+\sigma_{hhhh} \cos^2\alpha \sin^2\alpha + \sigma_{hhhv} \cos\alpha \sin^3\alpha \\ &- \sigma_{hhvh} \cos^3\alpha \sin\alpha - \sigma_{hhvv} \cos^2\alpha \sin^2\alpha \\ &+ \sigma_{hvhh} \cos\alpha \sin^3\alpha + \sigma_{hvhv} \sin^4\alpha \\ &- \sigma_{hvvh} \cos^2\alpha \sin^2\alpha - \sigma_{hvvv} \cos\alpha \sin^3\alpha \\ &- \sigma_{vhhh} \cos^3\alpha \sin\alpha - \sigma_{vhhv} \cos^2\alpha \sin^2\alpha \\ &+ \sigma_{vhvh} \cos^4\alpha + \sigma_{vhvv} \cos^3\alpha \sin\alpha \\ &- \sigma_{vvhh} \cos^2 \sin^2\alpha - \sigma_{vvhv} \cos\alpha \sin^3\alpha \\ &+ \sigma_{vvvh} \cos^3\alpha \sin\alpha + \sigma_{vvvv} \cos^2\alpha \sin^2\alpha \end{aligned} \tag{A8}$$

$$\begin{aligned}\sigma'_{vhvv} = &-\sigma_{hhhh} \cos\alpha \sin^3\alpha + \sigma_{hhhv} \cos^2\alpha \sin^2\alpha \\ &+ \sigma_{hhvh} \cos^2\alpha \sin^2\alpha - \sigma_{hhvv} \cos^3\alpha \sin\alpha \\ &- \sigma_{hvhh} \sin^4\alpha + \sigma_{hvhv} \cos\alpha \sin^3\alpha \\ &+ \sigma_{hvvh} \cos\alpha \sin^3\alpha - \sigma_{hvvv} \cos^2\alpha \sin^2\alpha \\ &+ \sigma_{vhhh} \cos^2\alpha \sin^2\alpha - \sigma_{vhhv} \cos^3\alpha \sin\alpha \\ &- \sigma_{vhvh} \cos^3\alpha \sin\alpha + \sigma_{vhvv} \cos^4\alpha \\ &+ \sigma_{vvhh} \cos\alpha \sin^3\alpha - \sigma_{vvhv} \cos^2\alpha \sin^2\alpha \\ &- \sigma_{vvvh} \cos^2\alpha \sin^2\alpha + \sigma_{vvvv} \cos^3\alpha \sin\alpha \end{aligned} \tag{A9}$$

$$\begin{aligned}\sigma'_{vvvv} = &+\sigma_{hhhh} \sin^4\alpha - \sigma_{hhhv} \cos\alpha \sin^3\alpha \\ &- \sigma_{hhvh} \cos\alpha \sin^3\alpha + \sigma_{hhvv} \cos^2\alpha \sin^2\alpha \\ &- \sigma_{hvhh} \cos\alpha \sin^3\alpha + \sigma_{hvhv} \cos^2\alpha \sin^2\alpha \\ &+ \sigma_{hvvh} \cos^2\alpha \sin^2\alpha - \sigma_{hvvv} \cos^3\alpha \sin\alpha \\ &- \sigma_{vhhh} \cos\alpha \sin^3\alpha + \sigma_{vhhv} \cos^2\alpha \sin^2\alpha \\ &+ \sigma_{vhvh} \cos^2\alpha \sin^2\alpha - \sigma_{vhvv} \cos^3\alpha \sin\alpha \\ &+ \sigma_{vvhh} \cos^2 \sin^2\alpha - \sigma_{vvhv} \cos^3\alpha \sin\alpha \\ &- \sigma_{vvvh} \cos^3\alpha \sin\alpha + \sigma_{vvvv} \cos^4\alpha \end{aligned} \tag{A10}$$

$$\sigma'_{hvhh} = \sigma'^{*}_{hhhv}, \quad \sigma'_{vhhh} = \sigma'^{*}_{hhvh}, \quad \sigma'_{vhhv} = \sigma'^{*}_{hvvh} \tag{A11}$$

$$\sigma'_{vvhh} = \sigma'^{*}_{hhvv}, \quad \sigma'_{vvhv} = \sigma'^{*}_{hvvv}, \quad \sigma'_{vvvh} = \sigma'^{*}_{vhvv} \tag{A12}$$

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